

# “Bimaximal + Democratic” type neutrino mass matrix in view of quark-lepton complementarity

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We demonstrate that ‘Bimaximal + Democratic’ type neutrino mass matrix can accommodate the deviation of  $\theta_\odot$  from its maximal value which referred in the literature as ‘quark-lepton complementarity’ along with the other present day neutrino experimental results, namely, atmospheric, CHOOZ, neutrinoless double beta decay ( $\beta\beta_{0\nu}$ ) and result obtained from WMAP experiment. We define a function  $\chi_p$  in terms of solar and atmospheric neutrino mass squared differences and solar neutrino mixing angle (obtained from different experiments and our proposed texture). The masses and mixing angles are expressed in terms of three parameters in our proposed texture. The allowed region of the texture parameters is obtained through minimization of the above function. The proposed texture crucially depends on the value of the experimental results of  $\beta\beta_{0\nu}$  experiment among all other above mentioned experiments. If, in future,  $\beta\beta_{0\nu}$  experiments, namely, MOON, EXO, GENIUS shift the lower bound on  $\langle m_{ee} \rangle$  at the higher side by one order, the present texture will be ruled out.

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## I. INTRODUCTION

The conjecture of neutrino flavor oscillation has been strengthened by the Super-Kamiokande (SK) [1] atmospheric neutrino experiment. This has also been substantiated by K2K long baseline neutrino experimental results [2]. The best-fitted values obtained from SK experiment are given by  $\Delta m_{\text{atm}}^2 = 2.4 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{\text{atm}} = 1.0$  [3]. Moreover, global analysis of all solar neutrino experimental results, including SNO salt phase experiment, [4] is in favor of Large Mixing Angle (LMA) MSW solution of solar  $\nu_e$  deficit problem and this has also in concordance with the KamLAND [5] reactor neutrino experimental results. Furthermore, the CHOOZ [6] experiment has also put an upper bound on the  $U_{e3}$  element of the lepton mixing matrix and combined analysis of solar, atmospheric and reactor data put bound on  $U_{e3}$  as  $|U_{e3}| \leq 0.22$  at 99.73% c.l [7]. There are two other experimental results, one of them, from WMAP experiment [8] on cosmic microwave background anisotropies gives an upper bound on the total neutrino mass as  $\Sigma m_i \leq 0.63 \text{ eV}$  [9]. The second one, the neutrinoless double beta experiment [10] has put a bound on Majorana-type neutrino mass as  $\langle m_{ee} \rangle = (0.05 - 0.84) \text{ eV}$  at 99% c.l with an uncertainty of the nuclear matrix elements up to 50%.

In view of the above it is worthwhile to investigate appropriate texture of lepton mass matrix which satisfies all those experimental results. First of all, exact ‘bimaximal’ texture is not admissible due to departure of  $\theta_\odot$  from  $\pi/4$  and the deviation is parametrised in terms of Cabibbo angle and the phenomena is referred as ‘quark-lepton complementarity’ [11]. Keeping the charged lep-

ton mass matrix flavor diagonal, in the present work, we consider a neutrino mass matrix of the form ‘bimaximal + democratic’ where the ‘democratic’ part of the neutrino mass matrix parametrise the difference  $\pi/4 - \theta_\odot$ . We have not addressed any typical model in this work, because, there are plenty of literature concerning bimaximal structure of neutrino mass matrix, however, we have indicated possible source of democratic type matrix explicit realization of which needs some extra flavor symmetry. The present texture gives rise to  $\theta_{\text{atm}}$  maximal and vanishing value  $\theta_{13}$ . We fit two neutrino mass-squared differences with the required values of solar and atmospheric neutrino experimental results and the remaining mixing angle with the required values of solar neutrino neutrino mixing angle. These three parameters are fixed by defining a function  $\chi_p^2$  (see later) as the sum of squares of the differences between the calculated values of neutrino oscillation parameters (with the texture considered) and the best fitted values of the same (obtained from different analyses) and then minimising this function. The plan of paper is as follows: In Section II, we propose the texture of the neutrino mass matrix and its eigenvalues and mixing angles. We discuss phenomenological fits of the model parameters through our defined  $\chi_p^2$  function in Section III. Section IV contains our conclusion.

## II. PROPOSED TEXTURE

We consider the following texture of the neutrino mass matrix

$$M_\nu = M_{bi} + M_{demo} \quad (2.1)$$

where the first part  $M_{bi}$  is given by

$$M_{bi} = \begin{pmatrix} 0 & a' & a' \\ a' & 0 & c' \\ a' & c' & 0 \end{pmatrix} \quad (2.2)$$

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and the second part  $M_{demo}$  is as

$$M_{demo} = d \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad (2.3)$$

where  $d$  is some scale factor. Thus the entire mass matrix contains three parameters  $a, c$  and  $d$  and we consider all of them to be real. A possible source of the terms which gives rise to ‘democratic’ type mass matrix is due to higher dimensional operator

$$\mathcal{L} = \frac{f_{ij}}{M} \sum_{i,j} l_{iL} l_{jL} \phi \phi \quad (2.4)$$

through considering equality between the  $f_{ij}$  couplings. Diagonalizing the entire neutrino mass matrix we obtain the following mixing angles

$$\theta_{23} = -\pi/4, \theta_{31} = 0 \quad (2.5)$$

$$\tan^2 \theta_{12} = \frac{d - m_1}{m_2 - d} \quad (2.6)$$

and the eigenvalues are

$$\begin{aligned} m_1 &= \frac{3d + c + x}{2} \\ m_2 &= \frac{3d + c - x}{2} \\ m_3 &= -c \end{aligned} \quad (2.7)$$

where

$$x = \sqrt{(c + d)^2 + 8(a + d)^2} \quad (2.8)$$

where  $a = a' + d$  and  $c = c' + d$ . Next, we set the solar and atmospheric neutrino mass squared differences as

$$\Delta m_{sol}^2 = \Delta m_{21}^2 = m_1^2 - m_2^2 = x(3d + c) \quad (2.9)$$

and

$$\Delta m_{atm}^2 = \Delta m_{23}^2 = m_2^2 - m_3^2 = (3(d + c) - x)(c - 3d + x)/4 \quad (2.10)$$

The best fit values of oscillation parameters from solar neutrino experiment, atmospheric neutrino experiments and solar neutrino mixing angle are used to obtain the values of  $a, c$  and  $d$ . Before going to the numerical fit of the parameters, we would like to mention that the present mass matrix contains three parameters which are fitted with three experimental values, hence, the texture has predictions about the neutrinoless double beta decay result and cosmological constraint on total neutrino mass (WMAP experimental result). The predictability of the texture would increase if we incorporate more symmetries in an explicit model, which necessarily reduce

the number of parameters. However, since the present texture admits nicely the constraint from WMAP experiment, the only testability of the present texture relies crucially on the future experimental results of neutrinoless double beta decay. If this experimental result gives a strong evidence in favour of Majorana-type neutrino mass, then the present texture should explain the result or it will be ruled out. In the next section we will fit those values with the best fit results through our defined  $\chi_p^2$  function.

### III. NEUTRINO PHENOMENOLOGY

The best fit values of oscillation parameters from solar neutrino experiment and atmospheric neutrino experiments are used to obtain the values of  $a, c$  and  $d$ . For this purpose, we consider  $\Delta m_{12}^2$ , the difference of the square of mass eigenstates  $m_1$  and  $m_2$  and the mixing angle  $\theta_{12}$  that are responsible for solar neutrino oscillation and  $\Delta m_{23}^2, \theta_{23}$  responsible for oscillation of atmospheric neutrinos. A recent global analysis [4] of the solar neutrino data from all solar neutrino experiments namely Chlorine, Gallium, Super-Kamiokande, SNO charged current, neutral current including salt phase data, shows that large mixing angle or LMA solution is most favoured for solar neutrino oscillation. From the combined analysis with solar neutrino data and KamLand experiment the best fit values of the solar neutrino oscillation parameters are given by  $\Delta m_{\odot}^2 (\equiv \Delta m_{12}^2 \text{ for our model}) = 8 \times 10^{-5} \text{ eV}^2$  and  $\tan^2 \theta_{\odot} (\equiv \tan^2 \theta_{12} \text{ for our model}) = 0.45$  [4]. From the analysis of SK atmospheric neutrino oscillation data, [3] we have the best fit values of  $\Delta m_{atm}^2 (\equiv \Delta m_{23}^2 \text{ for our model}) = 2.4 \times 10^{-3} \text{ eV}^2$  and  $\theta_{atm}$  is maximal. This value of  $\theta_{atm}$  has already been obtained in our proposed texture for  $\theta_{23}$ . Thus treating  $a, c$  and  $d$  as parameters we can obtain different values of  $\Delta m_{12}^2, \Delta m_{23}^2$  and  $\tan^2 \theta_{12}$  and compare them with best fit values of those quantities namely  $\Delta m_{\odot}^2, \Delta m_{atm}^2$  and  $\tan^2 \theta_{\odot}$  obtained from the analysis of solar and atmospheric neutrino data (discussed above) to fix  $a, c$  and  $d$ . To this end we define a function

$$\begin{aligned} \chi_p^2 &= (1 - \Delta m_{12}^2 / \Delta m_{\odot}^2)^2 + (1 - \Delta m_{23}^2 / \Delta m_{atm}^2)^2 \\ &+ (1 - \tan^2 \theta_{12} / \tan^2 \theta_{\odot})^2. \end{aligned} \quad (2.11)$$

The function  $\chi_p^2$  as defined above is calculated for a wide range of values of  $a, c$  and  $d$  and the minimum of the function is obtained. The corresponding values of  $a, c$  and  $d$  are given below.

$$\begin{aligned} a &= -0.145 \text{ eV} \\ c &= -0.152 \text{ eV} \\ d &= 0.051 \text{ eV} \end{aligned} \quad (2.12)$$

$\Delta m_{12}^2$ ,  $\Delta m_{23}^2$  and  $\tan^2 \theta_{12}$  obtained from the above values of  $a$ ,  $c$  and  $d$  and their comparison with the best fit values for  $\Delta m_{\odot}^2$ ,  $\Delta m_{\text{atm}}^2$  and  $\tan^2 \theta_{\odot}$  obtained from recent analysis of the solar and atmospheric neutrino data are shown in Table 1. In order to find out the range of values of  $a$ ,  $c$  and  $d$  that satisfy the  $1\sigma$  limits of  $\Delta m_{\odot}^2$  and  $\tan^2 \theta_{\odot}$  for the ([4]) combined analysis of recent solar neutrino data and KamLand data, we have fixed the value of  $\Delta m_{23}^2$  at its best fit value (shown in Table 1) and vary the parameters  $a$ ,  $c$  and  $d$  such that  $\Delta m_{12}^2$  and  $\tan^2 \theta_{12}$  satisfy the allowed range mentioned above. This range as given in Ref. [4] is  $7.6 \times 10^{-5} \text{eV}^2 < \Delta m_{\odot}^2 < 8.6 \times 10^{-5} \text{eV}^2$  and  $0.38 < \tan^2 \theta_{\odot} < 0.54$ . The allowed region in parameter space of  $a$ ,  $c$  and  $d$  are shown in a 3D plot (Fig. 1). As seen from Fig. 1, the allowed region is confined in a very narrow band in the parameter space. We have also found the parameter space in  $a$ ,  $c$  and  $d$  for which  $\Delta m_{23}^2$  lies in the 90% c.l. range given by  $2.0 \times 10^{-3} \text{eV}^2 \leq \Delta m_{23}^2 \leq 5.0 \times 10^{-3} \text{eV}^2$  while the other oscillation parameters kept in the previous range. This range of  $\Delta m_{23}^2$  with 90 % c.l for SK atmospheric neutrino data is obtained from [3]. The allowed region in the parameter space of  $a$ ,  $c$  and  $d$  are shown in the 3D plot of Fig. 2. The value of  $d$  gives rise to the value of  $M$  as

$$M = \langle \phi \rangle^2 / d \simeq 10^{15} \text{GeV}. \quad (2.13)$$

assuming  $\langle \phi \rangle = 250 \text{ GeV}$ . This value of  $M$  could be recognized as the right-handed symmetry breaking scale in some models which contains an extra  $U(1)_R$  or  $SU(2)_R$  symmetry in which neutrino masses are generated through see-saw mechanism. Again, the sum of the three neutrino mass comes out as  $\Sigma m_i = 0.44 \text{ eV}$  which is far below from the upper bound of WMAP experimental result. Furthermore, the value of  $d$  obtained from the

best fit result is at the lower end of the present limit on Majorana neutrino mass ( $\langle m_{ee} \rangle = 0.05 - 0.84 \text{eV}$  [10] obtained from the neutrinoless double beta decay experiment. Although the result is somewhat controversial [12], however, the testability of the present model lies crucially on the future experiments such as, EXO, MOON, GENIUS etc.

#### IV. CONCLUSION

We demonstrate that quark-lepton complementarity can be realised in a 'Bimaximal + democratic' type neutrino mass matrix which includes two possible sources of neutrino mass. One of them is the usual 'Bimaximal' type arise in different models and a possible source of 'Democratic' type is through the incorporation of higher dimensional mass terms assuming equality between yukawa couplings. We then fix the parameters through our defined  $\chi_p^2$  function with the two mass squared differences solar and atmospheric and with the solar neutrino mixing angle. The texture admits a best fit result at the minimum value of  $\chi_p^2$ . Furthermore, the texture gives rise to a slightly lower value of  $m_{ee}$  mass compare to the experimental result of neutrinoless double beta decay experiment, and if the result survives in future with MOON, EXO, GENIUS experimental results, the present texture will be ruled out.

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Present Work	Experiment
$\Delta m_{12}^2$ (eV <sup>2</sup> )	$\Delta m_{\odot}^2$ Ref.[4] (eV <sup>2</sup> )
$8.02 \times 10^{-5}$	$8.0 \times 10^{-5}$
$\Delta m_{23}^2$ (eV <sup>2</sup> )	$\Delta m_{\text{atm}}^2$ Ref.[3] (eV <sup>2</sup> )
$2.6 \times 10^{-3}$	$2.4 \times 10^{-3}$
$\tan^2 \theta_{12}$	$\tan^2 \theta_{\odot}$ Ref.[4]
0.476	0.45

TABLE I: Best fitted values for neutrino oscillation parameters obtained in the present work.

### Figure Caption

Fig. 1 The 3D plot showing the region of the parameters  $a$ ,  $c$  and  $d$  that produce the values of  $\Delta m_{12}^2$  and  $\tan^2 \theta_{12}$  within  $1\sigma$  range of the best fitted values from global solar neutrino + KamLand data analysis [4].  $\Delta m_{23}^2$  remains fixed at the best fit value. See text for details.

Fig. 2 Same as Fig. 1 but in this case instead of keeping  $\Delta m_{23}^2$  fixed at the best fit value, it is varied within the 90% C.L. range  $2 \times 10^{-3} < \Delta m_{23}^2 < 5 \times 10^{-3} \text{eV}^2$  [3].



